Estimates of B-Decays into K-Resonances and Dileptons

Mohammad R. Ahmady
Department of Physics, Ochanomizu University
1-1, Otsuka 2, Bunkyo-ku, Tokyo 112, Japan

Dongsheng Liu
Department of Physics, University of Tasmania, Hobart, Australia 7001

Amir H. Fariborz
Department of Applied Mathematics, The University of Western Ontario
London, Ontario, Canada N6A 5B7

ABSTRACT

Short and long distance contributions to the exclusive B-decays into various K-resonances and dileptons, i.e. $B \to K^i\ell\bar{\ell}(\ell = e, \mu, \nu)$, are examined. The heavy quark effective theory has been used to calculate the hadronic matrix elements. Substantial branching fractions are obtained for the dileptonic B-decays into some higher excited states of K-mesons. The long distance (resonance) contributions to these exclusive rare B-decay modes dominate the short distance contributions mostly by two orders of magnitude. It is pointed out that, excluding the resonance contributions, the P-wave channels are dominant, accounting for about 50% of the inclusive $B \to X_s\ell^+\ell^-$ branching fraction.

1 Introduction

Rare B-decays have been in the focus of a lot of theoretical attentions . This is due to the extensive amount of information on Standard Model (SM) that can be extracted from these processes. The rare decays proceed through flavor changing neutral current (FCNC) vertices that are absent at the tree level, therefore, they are a good probe of the SM at the quantum (loop) level. On the other hand, radiative B-decays are sensitive to quark mixing angles V_{td}, V_{ts} and V_{tb}, hence their measurements yield valuable information on CKM matrix elements, and consequently, shed some light on the CP violation in SM. Also, reducing the uncertainties of the theoretical calculation of these processes within the SM, and comparing them with more precise measurements available at future B-factories, may be the most promising probe of the new physics in the short term.

In this paper, we focus on the rare B-decays, $B \to K^i \ell\bar{\ell}(\ell = e, \mu, \nu)$, where K^i are various resonances of K-meson. We calculate both, the short-distance (SD), and the long-distance (LD) contributions to these processes. The motivation for this work is the following: Due to the large phase space available to rare B-decays, exclusive processes with excited K-mesons in the final state might have substantial branching fractions. In fact, in reference [1], it is shown that for the rare B-decays $B \to K^i\gamma$, which proceed through SD penguin operator, some higher resonances of K-meson have 2-3 times larger branching ratios than the ground state. On the other hand, in reference [2], we indicated that the LD contributions to these processes are very significant, approximately 20% of the SD amplitude. Consequently, we feel that the same investigation should be carried out on dileptonic rare B-decays.

A main source of uncertainty, in all these estimates, has always been the evaluation of hadronic matrix elements (HME) for specific exclusive decays. This would involve the long range nonperturbative QCD effects which renders the calculations model-dependence. We use the heavy quark effective theory (HQET)[3], as a convenient ansatz in formulating the HME. This results in an enormous simplification, especially when decay rates to higher excited K-meson states are estimated. With a plausible assumption that **b** quark is static within the B meson, one can show that these HME are the same as those appearing in $B \to K^i\psi(\psi')$ decays. Therefore, one can adjust the universal form factor, such that a HQET formulation of the HME fits the experimental data available for $B \to K^{(*)}\psi^{(\prime)}$[4]. In fact, our predicted value for the ratio $R = \Gamma(B \to K^*\gamma)/\Gamma(B \to X_s\gamma)$, obtained this way (see eqn. (22) in Ref. [4]), is in good agreement with the recent CLEO results [6]. In a follow up paper, we showed, by adopting a more general model for the universal form factors, one can also estimate the non-leptonic decays to higher K-meson resonances [5]. These predictions are yet to be tested experimentally.

2 Differential decay rates

We start with the effective Lagrangian for $b \to s\ell^+\ell^-$, which includes both, short and long distance contributions [7, 8]

$$L_{eff} = \frac{G_F}{\sqrt{2}} \left(\frac{\alpha}{4\pi s_W^2} \right) V_{ts}^* V_{tb} (A \bar{s} L_\mu b \bar{\ell} L^\mu \ell + B \bar{s} L_\mu b \bar{\ell} R^\mu \ell + 2 m_b s_W^2 F \bar{s} T_\mu b \bar{\ell} \gamma^\mu \ell), \qquad (1)$$

where

$$L_\mu = \gamma_\mu (1 - \gamma_5), \quad R_\mu = \gamma_\mu (1 + \gamma_5),$$

and

$$T_\mu = -i \sigma_{\mu\nu} (1 + \gamma_5) q^\nu / q^2.$$

V_{ij} are the Cabibbo-Kobayashi-Maskawa matrix elements, $s_W^2 = sin^2\theta_W \approx 0.23$ (θ_W is the weak angle), G_F is the Fermi constant and q is the total momentum of the final $\ell^+\ell^-$ pair.

This effective Lagrangian is obtained by integrating out the heavy degrees of freedom, ie. the top quark, W and Z bosons, at the scale $\mu = M_W$. Using the renormalization group equations to scale down to a subtraction point comparable to the light masses, ie. $\mu \approx m_b$, ensures that large logarithms, $ln(M_W/m_b)$, are contained in the coefficient functions A, B and F only. On the other hand, these functions, which their explicit forms can be found elsewhere [7, 8], depend on the top quark mass. For our numerical evaluations we use $m_t = 180$ GeV, which is the weighted average of the recent CDF and D0 results [9].

The terms representing the long distance contributions to the decay rates, enter coefficients A and B from the one-loop (charm) matrix element of the four-quark operators. Besides $c\bar{c}$ continuum (free quark) contribution, these coefficients receive the pole contributions from the J/ψ and ψ', which due to their narrow widths, amount to Breit-Wigner terms for these resonances.

The HME of (1) are calculated by using the trace formalism in the context of HQET [10]:

$$< K^i(v') | \bar{s} \Gamma b | B(v) > = Tr \left[\bar{R}^i(v') \Gamma R(v) M(v, v') \right]. \qquad (2)$$

$R^i(v')$ and $R(v)$ are the matrix representations of K^i and B respectively, v and v' are velocities of initial and final state mesons. M, which represents the light degrees of freedom, is related to the Isgur-Wise functions.

We classify various K-resonances into spin doublets [1, 5], and thus, using (2), the HME to each doublet is expressed in terms of a single universal function. These functions represent the underlying non-perturbative QCD dynamics, which at present is not possible to calculate from first principles and certain model assumptions are required for their evaluation.

We obtain the following expression for the differential decay rates in the limit of massless leptons:

$$\begin{aligned}
\frac{d\Gamma(B\to K^i\ell^+\ell^-)}{dz} &= \frac{G_F^2}{96\pi^3}\left(\frac{\alpha}{4\pi s_W^2}\right)^2 M_B^5 y^2 (x_+x_-)^{1/2}|V_{ts}^*V_{tb}|^2|\xi_I(x)|^2 \\
&\times \Big((|A|^2+|B|^2)F_i + 2|C|^2 G_i + 2Re[(A+B)^* C]H_i\Big)\,, \qquad (3)\\
& x = v\cdot v',\ x_\pm = x\pm 1,\ y=\frac{m_{K^i}}{m_B},\ z=\frac{q^2}{m_B^2},\ C = s_W^2 F,
\end{aligned}$$

where $\xi_I(x), (I = C, E, F$ and $G)$, are the Isgur-Wise functions for each spin doublet [1, 5], and q^2 is the invariant mass of the dilepton. The functions F_i, G_i and H_i for various K-resonances are tabulated in Table 1. For $m_t = 180$ GeV, $\alpha_s(M_W) = 0.118$, and $\eta = \alpha_s(\mu)/\alpha_s(M_W)$ for these processes taken to be 1.75, we obtain:

$$\begin{aligned}
A &= 2.020 + (\text{continuum} + \text{resonance terms}),\\
B &= -0.173 + (\text{continuum} + \text{resonance terms}),\\
C &= -0.146.
\end{aligned}$$

This shows, excluding the charm loop contributions, the coefficient A is much larger than B or C for a large top quark mass.

Now we turn to the processes $B\to K^i\nu\bar\nu$, which even though are difficult to measure experimentally, can be tagged by a large missing energy-momentum. The effective Hamiltonian for $b\to s\nu\bar\nu$, arises from the coupling of pure $V-A$ currents,

$$H_{eff} = -\frac{G_F}{\sqrt{2}}\left(\frac{\alpha}{4\pi s_W^2}\right) V_{ts}^*V_{tb}[2E(t)](\bar s L_\mu b)(\bar\nu L^\mu \nu), \qquad (4)$$

where $t = m_t^2/m_W^2$. The Wilson coefficient $E(t)$ (see ref. [11] for the explicit form), is not affected by QCD scaling corrections. We take $E(t) = 1.66$ for $m_t = 180$ GeV. Using (2), we obtain the following differential decay rates for $B\to K^i\nu\bar\nu$

$$\frac{d\Gamma(B\to K^i\nu\bar\nu)}{dz} = \frac{G_F^2}{96\pi^3}\left(\frac{\alpha}{4\pi s_W^2}\right)^2 M_B^5 y^2(x_+x_-)^{1/2}|V_{ts}^*V_{tb}|^2|\xi_I(x)|^2|2E(t)|^2(3F_i). \qquad (5)$$

The factor 3 mutiplying F_i in (5), is due to the sum over neutrino flavors.

3 Isgur-Wise function and total decay rates

In order to estimate the total decay rates, $\Gamma(B\to K^i\ell\bar\ell)$, we need to insert the Isgur-Wise functions, $\xi_I(x)$, in (3) and (5). As we mentioned earlier, in doing so, we need to adopt a

model that could also be extended to higher excited final states. The wavefunction model of Isgur, Wise, Scora and Grinstein [12], suits our purpose quite well. The functions ξ_I, in this model, can be obtained from the overlap integrals:

$$\xi(v.v') = \sqrt{2L+1}\, i^L \int r^2 dr \Phi_F^*(r)\Phi_I(r) j_L\left[\Lambda r\sqrt{(v.v')^2 - 1}\right]. \tag{6}$$

I and F are initial and final radial wavefunctions respectively, L is the orbital angular momentum of the final state meson, and j_L is the spherical Bessel function of order L. The inertia parameter Λ, is taken to be [13]

$$\Lambda = \frac{m_{K^i} m_q}{m_s + m_q},$$

where m_q is the light quark mass.

We follow IWSG model in using harmonic oscillator wavefunctions with oscillator strength β for the radial wavefunctions Φ_I and Φ_F. β is assumed to be the same for initial and final state mesons in order to satisfy the normalization condition $\xi_C(1) = 1$ and $\xi_{E,F,G}(1) = 0$. For example, one obtains

$$\xi_C(v.v') = exp\left[\frac{9}{256\beta^2} m_{K^i}^2 (1 - (v.v')^2)\right] \tag{7}$$

for the $(0^-, 1^-)$ spin doublet by using the ground state radial wavefunction. $\beta = 0.295 GeV$ is fixed by the best fit of (7) to the experimentally measured $B \to (K, K^*) + (\psi, \psi')$ decays [14, 5]. We use the same value for β in our calculation of ξ_E, ξ_F, andξ_G.

We would like to remark that eqn. (7) implies the dependence of the universal form factor on the K-meson mass. In the heavy quark limit, the members of a spin doublet are degenerate in mass, which results in exactly one form factor for each doublet. However, for K-mesons, insertion of a realistic value for the mass parameter in (7), leads to a different form factor for each member of a doublet. This can be thought of as a relaxation of the spin symmetry to some extent which, in our view, results in a more reliable estimate of $B \to K^i$ transitions.

The following values are used for our numerical estimates:

$$\begin{array}{ll} m_B = 5.28\text{GeV} & m_c = 1.50\text{GeV}, \\ m_b = 4.95\text{GeV} & |V_{ts}| = 0.043. \end{array}$$

The partial decay widths to various K-resonances are tabulated in Table 2. For $B \to K^i e^+ e^-$ and $B \to K^i \mu^+ \mu^-$, these partial decay rates are different only in the first interval, ie. small invariant dilepton mass region, where the lower bound of the interval, $4m_\ell^2$, is determined by the lepton mass. This difference is shown in the first and second columns of Table 2. The enteries of the 3rd, 4th, 5th and 6th columns of that table

are the partial decay widths for $B \to K^i \ell^+ \ell^-$ ($\ell = e\ ,\mu$) over $\left((m_\psi - \delta)^2\ ,(m_\psi + \delta)^2\right)$, $\left((m_\psi + \delta)^2\ ,(m_{\psi'} - \delta)^2\right)$, $\left((m_{\psi'} - \delta)^2\ ,(m_{\psi'} + \delta)^2\right)$, and $\left((m_{\psi'} + \delta)^2\ ,(m_B - m_{K^i})^2\right)$ respectively. We choose $\delta = 0.2$ GeV for our numerical estimates. For $K^*(1680)$ and $K_2(1580)$, $q^2_{\rm max} = (m_B - m_{K^i})^2$ is in fact smaller than $(m_{\psi'} - \delta)^2$ and $(m_{\psi'} + \delta)^2$ respectively.

For the decays to the ground state spin doublet ($K\ ,\ K^*$), we have also examined some other parametrizations of the Isgur-Wise function which are commonly used in the literature. These are the monopole form,

$$\xi(v.v') = \frac{\omega_\circ^2}{\omega_\circ^2 - 2 + 2v.v'},$$

with $\omega_\circ = 1.8$, and the exponential form,

$$\xi(v.v') = exp\left[\gamma(1 - v.v')\right],$$

with $\gamma = 0.5$. The parameter values are obtained from the best fit to the data on $D \to K\ell\nu_\ell$.

In Table 3, we compare the total branching fractions (excluding the resonance contributions) when different Isgur-Wise functions are used for dileptonic B-decays to K and $K^*(892)$. The total width of the B is taken to be $\Gamma(B \to \text{all}) = 5.6 \times 10^{-13}$ GeV.

From (1) and (4), one can calculate the inclusive differential decay rates, ie.

$$\begin{aligned} \frac{1}{\Gamma(B \to X_c e\bar{\nu})}\frac{d\Gamma}{dz}(B \to X_s \ell^+\ell^-) = & \left(\frac{\alpha}{4\pi s_W^2}\right)^2 \frac{2}{f(m_c/m_b)} \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2}(1-z)^2 \\ \times & \Big((|A|^2 + |B|^2)(1+2z) + 2|C|^2(1+2/z) \\ & +6Re[(A+B)^* C]\Big) \end{aligned} \tag{8}$$

and

$$\begin{aligned} \frac{1}{\Gamma(B \to X_c e\bar{\nu})}\frac{d\Gamma}{dz}(B \to X_s \nu\bar{\nu}) = & \left(\frac{\alpha}{4\pi s_W^2}\right)^2 \frac{2}{f(m_c/m_b)} \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2}|2E(t)|^2 \times 3 \\ \times & \left(1 - 3z^2 + 2z^3\right), \end{aligned} \tag{9}$$

where

$$f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 ln(x).$$

Again, the factor 3 in (9) is due to sum over three neutrino flavors. By normalizing to the semileptonic rate in (8) and (9), the strong dependence on the b-quark mass cancels out. Using the above expressions, and assuming $|V_{cb}| \approx |V_{ts}|$, we obtain:

$$\begin{aligned} \Gamma(B \to X_s e^+e^-) &= 4.15 \times 10^{-18}\text{GeV}, \\ \Gamma(B \to X_s \mu^+\mu^-) &= 2.81 \times 10^{-18}\text{GeV}, \\ \Gamma(B \to X_s \nu\bar{\nu}) &= 2.43 \times 10^{-17}\text{GeV}, \end{aligned} \tag{10}$$

where the semileptonic branching ratio is taken to be $BR(B \to X_c e\bar{\nu}) = 0.105$.

In Table 4, we have tabulated the total decay rates (excluding the resonance contributions) and the ratio $R = \Gamma(B \to K^i \ell\bar{\ell})/\Gamma(B \to X_s \ell\bar{\ell})$ for various K-resonances.

4 Concluding remarks

We end this paper with a few concluding remarks.

From Table 2, we observe that the partial decay widths over the ψ resonance region (the 3$^{\rm rd}$ column) are about two orders of magnitude larger than the short distance contributions which are mainly from the first region (the 1$^{\rm st}$ and 2$^{\rm nd}$ columns for e and μ respectively). The exceptions are the d-wave transitions where this dominance is reduced to roughly one order of magnitude. As a result, the total decay rates, $\Gamma(B \to K^i \ell^+\ell^-)$, are dominated by the long distance (resonance) contributions. The partial decay widths over the ψ' resonance region (the 5$^{\rm th}$ column) are not as large, and in fact, except for the ground state transitions, the rest are mostly comparable or smaller than the short distance contributions.

The most promising probe of the short distance operators, which are sensitive to the new physics, can be done by imposing an upper limit on the invariant mass of the dileptons, ie. q^2 (see eqn. (3)), well below m_ψ^2. As it is reflected in the first and second columns of Table 2, for some final state K-resonances, there is a significant difference in this partial decay width between e^+e^- and $\mu^+\mu^-$ pair. This difference is overshadowed in the total decay rate by the overwhelming long distance contribution which is independent of the dilepton type. On the other hand, much larger partial decay widths is observed for some higher excited K-meson states if the above upper cut on q^2 is implemented. For example, for $q^2 \leq (m_\psi - \delta)^2$, $\Gamma(B \to K_2^*(1430)e^+e^-)$ and $\Gamma(B \to K_2^*(1430)\mu^+\mu^-)$ are larger than $\Gamma(B \to K^* e^+e^-)$ and $\Gamma(B \to K^*\mu^+\mu^-)$ by factors of 6 and 4 respectively (see the 1$^{\rm st}$ and 2$^{\rm nd}$ columns of Table 2). As we mentioned in the introduction, this is not unexpected, as there is a large mass difference between the initial B-meson and the final state K-mesons. Taking into account the efficiency for the reconstruction of the decaying K-mesons [15], our estimates in this work indicate that along with $K^*(892)$ and $K_2^*(1430)$, two other P-wave channels, ie. $K_1(1270)$ and $K_1(1400)$, are the best possible modes to look for rare dileptonic B-decays in the future B-factories.

We have compared our results for the ground state K-mesons, K and K^*, with those obtained from different parametrizations of the Isgur-Wise function. The branching fractions, tabulated in Table 3, indicate that, in general, the monopole form of the Isgur-Wise function results in a larger estimate of these decay modes. We should also point out that the smaller difference between $BR(B \to K^* e^+e^-)$ and $BR(B \to K^*\mu^+\mu^-)$ in IWSG model, is the direct result of the large recoil suppression of the Isgur-Wise function in this model.

Finally, we note from Table 4, that the exclusive channels we consider in this paper, more or less saturate the dileptonic rare B-decay modes.

Acknowledgement
The authors thank A. Ali, T. Zhijian and I. Watanabe for useful discussions. M.R.A. would like to acknowledge support from the Japanese Society for the Promotion of Science.

References

[1] A. Ali, T. Ohl and T. Mannel, Phys. Lett. B298 (1993) 195.

[2] M. R. Ahmady, Dongsheng Liu and Zhijian Tao, I.C.T.P. preprint IC/93/26.

[3] N. Isgur and M. Wise, Phys. Lett. B232 (1989) 113; Phys. Lett. B237 (1990) 527.

[4] M. R. Ahmady and D. Liu, Phys. Lett. B302 (1993) 491.

[5] M. R. Ahmady and D. Liu, Phys. Lett. B324 (1994) 231.

[6] R. Ammar et al., (CLEO Collaboration), Phys. Rev. Lett. 71 (1993) 674; M. S. Alam et al., (CLEO Collaboration), Phys. Rev. Lett. 74 (1995) 2885.

[7] B. Grinstein, M. J. Savage and M. B. Wise, Nucl. Phys. B319 (1989) 271.

[8] A. Ali, T. Mannel and T. Morozumi, Phys. Lett. B273 (1991) 505.

[9] F. Abe *et al.*, (CDF Collaboration), Phys. Rev. Lett. 74 (1995) 2626; S. Abachi *et al.*, (D0 Collaboration), Phys. Rev. Lett. (1995) 2632.

[10] A. Falk, Nucl. Phys. B378 (1992) 79.

[11] W. Hou, R. Willey and A. Soni, Phys. Rev. Lett. 58 (1987) 1608.

[12] B. Grinstein, N. Isgur, D. Scora and M. Wise, Phys. Rev. D39 (1989) 799.

[13] T. Altomari, Phys. Rev. D37 (1988) 677.

[14] T.E. Browder, *et al.*, (CLEO Collaboration), CLNS-93-1226.

[15] H. Albrecht *et al.*, (ARGUS Collaboration), Phys. Lett. B299 (1989) 304.

K^i Name	J^P	F^i	G^i	H^i
K	0^-	$x_+x_-(1+y)^2$	x_+x_-	$x_+x_-(1+y)$
$K^*(892)$	1^-	$x_+[x_+(1-y)^2+4xz]$	$x_+[x_++4/z((1+y)^2x_-+z)]$	$x_+[3(1-y)x_++2(1+y)x_-]$
$K^*(1430)$	0^+	$x_+x_-(1-y)^2$	x_+x_-	$x_+x_-(1-y)$
$K_1(1270)$	1^+	$x_-[x_-(1+y)^2+4xz]$	$x_-[x_-+4/z((1-y)^2x_+-z)]$	$x_-[3(1+y)x_-+2(1-y)x_+]$
$K_1(1400)$	1^+	$2/3x_-{x_+}^2[x_+(1-y)^2+xz]$	$2/3x_-{x_+}^2[x_-+1/z((1-y)^2x_++z)]$	$2/3x_-{x_+}^2[x_-(1+y)+x_-]$
$K_2^*(1430)$	2^+	$2/3x_-{x_+}^2[x_+(1-y)^2+3xz]$	$2/3x_-{x_+}^2[x_++3/z((1-y)^2x_+-z)]$	$2/3x_-{x_+}^2[x_+(1-y)+3x_-]$
$K^*(1680)$	1^-	$2/3x_+{x_-}^2[x_-(1+y)^2+xz]$	$2/3x_+{x_-}^2[x_++1/z((1+y)^2x_--z)]$	$2/3x_+{x_-}^2[x_+(1-y)+x_-]$
$K_2(1580)$	2^-	$2/3x_+{x_-}^2[x_-(1+y)^2+3xz]$	$2/3x_+{x_-}^2[x_-+3/z((1+y)^2x_-+z)]$	$2/3x_+{x_-}^2[x_-(1+y)+3x_-]$
$K(1460)$	0^-	$x_+x_-(1+y)^2$	x_+x_-	$x_+x_-(1+y)$
$K^*(1410)$	1^-	$x_+[x_+(1-y)^2+4xz]$	$x_+[x_++4/z(((1+y)^2x_-+z)]$	$x_+[3(1-y)x_++2(1+y)x_-]$

Table 1: The functions F_i, G_i and H_i for various K-resonances.

K^i Name	$4m_e^2,(m_\psi-\delta)^2$	$4m_\mu^2,(m_\psi-\delta)^2$	$(m_\psi-\delta)^2,(m_\psi+\delta)^2$	$(m_\psi+\delta)^2,(m_{\psi'}-\delta)^2$	$(m_{\psi'}-\delta)^2,(m_{\psi'}+\delta)^2$	$(m_{\psi'}+\delta)^2,q^2_{\max}$
K	1.1×10^{-19}	1.1×10^{-19}	3.3×10^{-17}	6.2×10^{-20}	2.4×10^{-18}	5.6×10^{-20}
$K^*(892)$	2.1×10^{-19}	1.9×10^{-19}	8.0×10^{-17}	1.8×10^{-19}	8.3×10^{-18}	1.8×10^{-19}
$K^*(1430)$	8.1×10^{-20}	8.0×10^{-20}	9.1×10^{-18}	6.6×10^{-21}	2.7×10^{-20}	3.2×10^{-23}
$K_1(1270)$	2.9×10^{-19}	1.8×10^{-19}	2.7×10^{-17}	2.4×10^{-20}	2.6×10^{-19}	7.3×10^{-22}
$K_1(1400)$	7.2×10^{-19}	5.7×10^{-19}	5.0×10^{-17}	3.8×10^{-20}	1.6×10^{-19}	2.3×10^{-22}
$K_2^*(1430)$	1.2×10^{-18}	7.5×10^{-19}	7.2×10^{-17}	5.3×10^{-20}	2.0×10^{-19}	2.3×10^{-22}
$K^*(1680)$	3.3×10^{-20}	2.1×10^{-20}	1.7×10^{-19}	5.8×10^{-23}	-	-
$K_2(1580)$	1.1×10^{-19}	5.7×10^{-20}	1.1×10^{-18}	4.4×10^{-22}	0	-
$K(1460)$	6.7×10^{-20}	6.7×10^{-20}	5.0×10^{-18}	2.8×10^{-21}	2.2×10^{-21}	1.3×10^{-24}
$K^*(1410)$	2.5×10^{-19}	1.8×10^{-19}	3.1×10^{-17}	2.5×10^{-20}	1.3×10^{-19}	1.7×10^{-22}

Table 2: Partial decay widths (in GeV) for $B\to K^i\ell^+\ell^-$ ($\ell=e,\mu$). The difference between electron and muon final states is in the first interval only (small invariant mass for dilepton pair) which is shown in the first two columns.

Isgur − Wise function	$B \to Ke^+e^-$	$B \to K\mu^+\mu^-$	$B \to K\nu\bar{\nu}$	$B \to K^*e^+e^-$	$B \to K^*\mu^+\mu^-$	$B \to K^*\nu\bar{\nu}$
Harmonic wavefunction $\beta = 0.295$	3.4×10^{-7}	3.4×10^{-7}	2.9×10^{-6}	8.9×10^{-7}	8.6×10^{-7}	8.0×10^{-6}
Monopole $\omega = 1.8$	6.6×10^{-7}	6.6×10^{-7}	5.7×10^{-6}	3.8×10^{-6}	2.5×10^{-6}	2.1×10^{-5}
Exponential $\gamma = 0.5$	2.9×10^{-7}	2.9×10^{-7}	2.5×10^{-6}	3.0×10^{-6}	2.1×10^{-6}	2.0×10^{-5}

Table 3: Comparison between the branching fractions of the rare B-decays to the ground state K-mesons (resonance contributions are excluded) for different Isgur-Wise functions.

K^i Name	J^P	Mass (MeV)	$B \to K^i e^- e^+$	$B \to K^i \mu^- \bar{\mu}^+$	$B \to K^i \nu\bar{\nu}$
K	0^-	497.67 ± 0.03	1.9×10^{-19} , (4.6)	1.9×10^{-19} , (6.8)	1.6×10^{-18} , (6.6)
$K^*(892)$	1^-	896.1 ± 0.3	5.0×10^{-19} , (12.0)	4.8×10^{-19} , (17.1)	4.5×10^{-18} , (18.5)
$K^*(1430)$	0^+	1429 ± 7	6.5×10^{-20} , (1.4)	6.5×10^{-20} , (2.3)	6.0×10^{-19} , (2.5)
$K_1(1270)$	1^+	1270 ± 10	2.7×10^{-19} , (6.5)	1.6×10^{-19} , (5.7)	1.3×10^{-18} , (5.3)
$K_1(1400)$	1^+	1402 ± 7	6.3×10^{-19} , (15.2)	4.8×10^{-19} , (17.1)	3.6×10^{-18} , (14.8)
$K_2^*(1430)$	2^+	1425.4 ± 1.3	1.1×10^{-18} , (26.5)	6.3×10^{-19} , (22.4)	4.5×10^{-18} , (18.5)
$K^*(1680)$	1^-	1714 ± 20	3.1×10^{-20} , (0.7)	1.9×10^{-20} , (0.7)	1.5×10^{-19} , (0.6)
$K_2(1580)$	2^-	≈ 1580	1.0×10^{-19} , (2.4)	5.1×10^{-20} , (1.8)	3.0×10^{-19} , (1.2)
$K(1460)$	0^-	≈ 1460	5.4×10^{-20} , (1.3)	5.4×10^{-20} , (1.9)	4.7×10^{-19} , (1.9)
$K^*(1410)$	1^-	1412 ± 12	2.2×10^{-19} , (5.3)	1.5×10^{-19} , (5.3)	1.3×10^{-18} , (5.3)
			Sum: 76.1%	Sum: 81.1%	Sum: 75.2%

Table 4: The total decay rates, in GeV (resonance contributions excluded), and the ratio $R = \Gamma(B \to K^i \ell\bar{\ell})/\Gamma(B \to X_s \ell\bar{\ell})$ (in %) for various K-resonances.